Thermal Control of a Dual Mode Parametric Sapphire Transducer

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***Abstract*—We propose a method to keep under control the thermal contributions to the stability in sapphire dielectric transducers, made by two dielectric disks separated by a thin gap and resonating in the whispering gallery (WG) modes of the electromagnetic field. Measuring at the same time the frequency of a WGH mode and a WGE mode allows to separate the information on gap and on temperature. A simple model, valid in quasi equilibrium conditions, describes the frequency shift of the two modes in terms of four tuning parameters. A procedure for the direct measurement of them is presented.**

I. Introduction

A dielectric whispering gallery resonator, made by two dielectric disks separated by a thin gap, can be a very sensitive displacement sensor. In particular, parametric microwave sapphire oscillators operating at cryogenic temperature has been proposed as readout systems for bar-type gravitational antennae [1] and also as core sensors for space-gravity tests and geodesy research [2]. Room-temperature operations of such devices demonstrated instead the possibility of high sensitive vibration measurements in very simple and robust setups [4].

The efficiency of this kind of transducer can be conveniently described by the merit factor $M = (Q \cdot \partial_z f)/f$, where Q is the resonator quality factor, f its resonance frequency and $\partial_z f$ is the tuning coefficient when gap spacing changes.

We are using here the classification of modes in WGH modes (characterized by E_z, H_ϕ, H_ρ) and WGE modes (with H_z, E_ϕ, E_ρ), where z denotes the field component along the disk rotation axis of the field vector, ρ the radial component, and ϕ the azimuthal component.

The most sensitive resonances has been demonstrated to be the WGH modes, with high order azimuthal number [1].

Even at room temperature, with sapphire disks of about 4 cm of diameter it is possible to observe a Q value of $\sim 10^5$ with a tuning factor of 6 $\mathrm{MHz}/\mu\mathrm{m}$ and $M = 60\ \mu\mathrm{m}^{-1}$. The relative frequency stability is however vanished by thermal effects, which contribute with some 10^{-5} K^{-1}. The strong temperature dependence of the dielectric tensor then compromises the long term stability.

On the other side, ultra-low frequency (i.e. daily timescale) displacement measurements have a key role in many applications such as gravimetric exploration, environmental monitoring and materials testing. The measurement of the resonant frequency of a parametric sapphire transducer contains a mixing of *pure-displacement* signals, *pure-temperature* signals and *temperature-induced displacement* signals, these last being given by the thermal expansion of the frame materials.

Due to the anisotropy of the crystal, the temperature coefficient of frequency is in general different for H and E modes, and exciting two different electromagnetic modes in the same resonator can allow to measure and stabilize the resonator temperature in oscillators [5], [6]. For a displacement sensor such an anisotropy affects also the tuning coefficient $\partial_z f$.
In this experimentral work we propose a calibration technique providing the estimation of pure displacement signals in a dual mode parametric sapphire resonator transducer operating at room temperature.

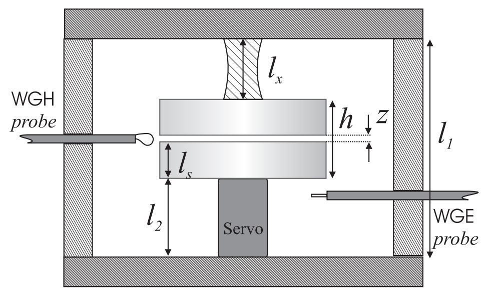

Fig. 1. A generic displacement sensor based on a transducer converting the physical quantity "'x"', in a vertical displacement δl_x. A servo actuator allows to control the separation set point. The sapphire disks are cut with the c-axis parallel to the cylinder axis.

II. Thermal effects in a Sapphire parametric transducer

The frequency dependence on temperature for the parametric displacement transducer reported in Fig.1 is described by the following equation :

$$\frac{1}{f}\frac{\partial f}{\partial T} = -p_{\epsilon\perp}\alpha_{\epsilon\perp} - p_{\epsilon||}\alpha_{\epsilon||} - p_D\alpha_D - p_h\alpha_h \quad (1)$$
$$-p_z(2\alpha_{l_s} + \alpha_{l_2} + \alpha_{l_X} - \alpha_{l_1})$$

where [6] for the generic quantity θ:

- $\alpha_\theta = \frac{1}{\theta}\frac{\partial\theta}{\partial T}$ are coefficients depending on the material properties; respectively: $\epsilon_\perp$ and $\epsilon_{||}$ the dielectric constants perpendicular and parallel to the c-axis; h and D the resonator spatial dimensions perpendicular and parallel to the c-axis; l_s, l_1, l_2, l_x the vertical dimensions of the single sapphire disk, the external frame, the lower support (actuator) and the upper support (moving part).

- $p_\theta = \frac{\theta}{f}\frac{\partial f}{\partial \theta}$ are the filling factors of the considered WG mode and depend on the field distribution of the mode inside the resonating volume.

For stationary conditions, one can assume the validity of a linear and time-indepedentent relation between a given pair of WG modes, the temperature T and the gap spacing z:

$$\begin{pmatrix} \delta f^{WGH} \\ \delta f^{WGE} \end{pmatrix} = \mathbf{C} \begin{pmatrix} \delta T \\ \delta z \end{pmatrix} \tag{2}$$

with

$$\mathbf{C} = \begin{pmatrix} C_T^{WGH} & C_z^{WGH} \\ C_T^{WGE} & C_z^{WGE} \end{pmatrix}. \tag{3}$$

The anisotropy of both the material and the field distribution ensures that it is possible to invert **C** so to obtain the frequency based estimation of the effective temparture fluctuations δT^* and pure (not thermally-induced) displacements δz^*:

$$\begin{pmatrix} \delta T^* \\ \delta z^* \end{pmatrix} = \mathbf{C}^{-1} \begin{pmatrix} \delta f^{WGH} \\ \delta f^{WGE} \end{pmatrix} \tag{4}$$

III. Calibration

The basis of the proposed method is to determine the coefficients of the matrix **C** by means of a calibration procedure. In Fig.2 is sketched the experimental apparatus for the sensor calibration. The sapphire transducer is placed inside a metallic chamber whose walls are temperature stabilized at about $34°$C. The chamber walls are electrically isolated one from each other in order to reduce sufficiently the disturbance of metal cavity modes on the choosen dielectric ones. A 4 cm thick insulation material covers the whole chamber to reduce the heat losses. The thermal control consists in a standard PI controller driving a set of thermoresistances in thermal contact with the metallic walls and the set point temperature can be driven by a voltage input for calibration purposes. The sapphire disks forming the resonator transducer are coupled to four microvaves antennae, two of them exciting the radial component of the electric field of a WGE mode, and the other two exciting the E_z and H_ϕ components of a WGH mode. The field distrubution of these two kind of modes allows to build up a double self-oscillating system where WGH and WGE modes oscillates simultaneously inside the the same resonator. We choose to test the system in the following conditions (see Fig.3):

$$\begin{aligned} f_{WGE_{11,1,1}} &\sim 11.38 \text{ GHz} \\ f_{WGH_{10,1,1}} &\sim 11.20 \text{ GHz}, \end{aligned} \tag{5}$$

for $z \sim 300$ μm. In this condition there is almost no interaction [3] between the selected modes and furthermore they are close enough in frequency so that it is possible to measure their beat note by means of an RF frequency counter. The resonator upper disk is rigidly fixed to the alluminium frame while the lower disk is mounted on a piezoelectric transducer.

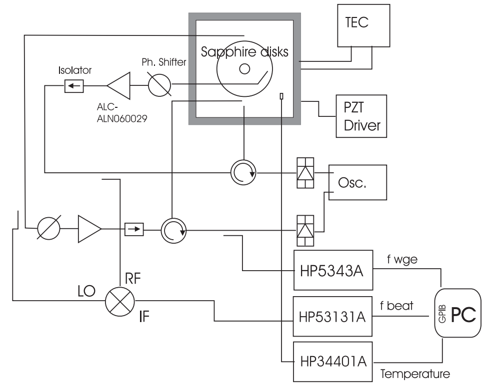

Fig. 2. Microwave circuitry for the dual mode self oscillating transducer suitable for the calibration.

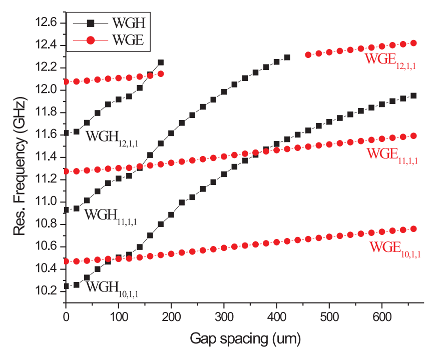

Fig. 3. Resonance frequencies of WGH and WGE modes versus gap spacing.

A PC based data acquisition system provides to the monitoring of the following physical quantities: internal temperature (resistance value of a Pt100 termistor), WGE mode frequency (microwave frequency counter), $WGE - WGH$ beat frequency (RF frequency counter).

IV. Demonstration of the Method

Once the dual frequency oscillation is set up, it is possible to evaluate the four coefficients of **C** by means of the temperature and position controllers. C_z^{WGE} and C_z^{WGH} are given by the ratio between the frequency variation of f^{WGE} and f^{WGH} and a calibrated gap variation induced by the piezo transducer moving the lower disk.

C_T^{WGE} and C_T^{WGH} are instead the proportionality constants between the frequency variation extrapolated at thermal equilibrium and the temperature variation induced by thermal actuators. In the present setup we obtained:
$C_T^{WGE} = 0.155$ MHz/°C, $C_T^{WGH} = 3.64$ MHz/°C,
$C_z^{WGE} = 0.567$ MHz/μm, $C_z^{WGH} = 3.43$ MHz/μm.

In Fig.4 is reported a comparison between δT^* and the temperature variation measured by a Pt100 thermometer placed inside the chamber. The measurement is referred to stabilized temperature conditions and rigid materials. The agreement

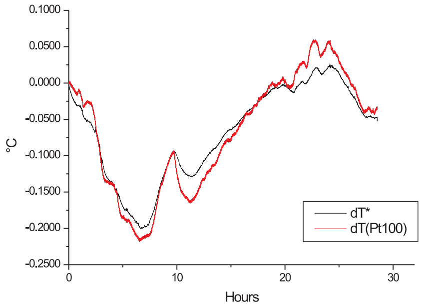

Fig. 4. Comparison between frequency based temperature evaluation and temperature measured by a Pt100 probe.

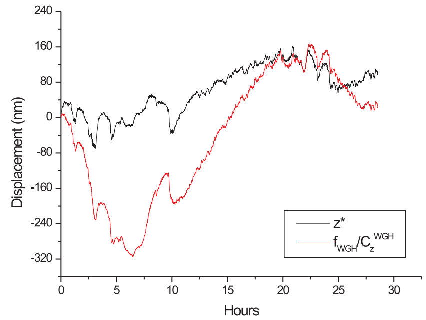

Fig. 5. Comparison between the dual frequency based displacement estimation and the estimation obtained from the single WGH mode.

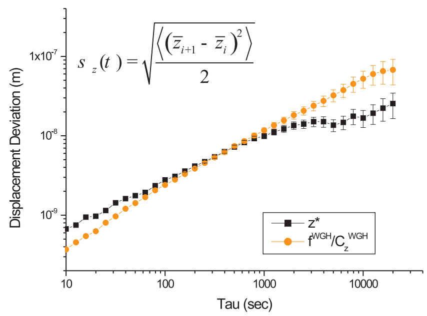

Fig. 6. Displacement deviation, see the inset formula, of the traces reported in Fig.5.

between the two traces confirm the validity of the model especially for very slow variations, giving the possibility of measuring the temperature of the sensor by means of frequency measurements.

In Fig.5 is reported a comparison between δz^* and f^{WGH}/C_z^{WGH} i.e. the displacement signal one would obtain with the measurement of the only f^{WGH} (the most sensitive of the two). Here it can be visible a net reduction of the correlation between displacement and temperature measurement.

It is worth noting that the hypothesis of thermal equilibrium for the system is fundamental for the effectivenes in the filtering out temperature noise from the displacement signal. In Fig.6 is sketched the comparison between the diplacement deviation for the two displacement traces. It can be seen that for integration time below about 200 sec the dual mode based displacement measurement is more noisy than the single frequency one. This is due to the fact that at this time scale the different sensor components are not in thermal equilibrium condition and the two frequencies are almost not correlated. For integration times above about 500 sec, a net reduction of noise can be observed for the trace relative to δz^*. This time scale correspond to the slowest of the time constants (sapphire thermalization) in the sensor: here thermal equilibrium approximation is almost valid and then the method provides the expected noise cancellation.

V. Conclusion

We proposed and tested an experimental technique for reducing the instabilities induced by thermal fluctuation in a sapphire parametric displacement transducer. Effective temperature of the sensor and pure displacement signal can be obtained from a double frequency measurement. The very simple time-independent model of the system has been tested on a trial setup in order to outline the potentiality and the limits of the method. As a prespective we foresee to improve the efficiency of the thermal stabilization and isolation system in order to enlarge the bandwidth of the noise filtering. Finally by implementing a double feedback-loop locking the two reconstructed quantities (δT^* and δz^*) it will be possible to work the system in static condition and then satisfying the model hypotesis at a better level of approximation.

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